

# HEAVY QUARKONIUM PHYSICS –THEORETICAL STATUS \*

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We briefly review the theoretical status and the open theoretical challenges in the physics of heavy quarkonium.

## 1. Interest of Heavy Quarkonium Physics

Systems made by two heavy quarks are particularly interesting from the theoretical point of view. They are characterized by the energy scales typical of a nonrelativistic bound system: the scale of the mass  $m$ , the scale of the relative momentum  $p \sim mv \sim r^{-1}$ , the scale of the binding energy  $E \sim mv^2$ ,  $v \ll 1$  being the quark velocity and  $r$  the radius of the system. This is similar to what happens for the hydrogen atom or for positronium in QED. The heavy quarks however interact strongly and their bound state dynamics is determined by QCD and subjected to confinement [1]. Besides the scales listed above, one has therefore to consider also  $\Lambda_{\text{QCD}}$ , the scale at which nonperturbative effects become important. The specific feature of being multi-scale makes heavy quarkonium an interesting probe for several energy regimes of QCD, from the hard region, where an expansion in the coupling constant  $\alpha_s$  is legitimate, to the low energy region, where QCD nonperturbative effects dominate. In particular the mass scale is “hard”,  $m \gg \Lambda_{\text{QCD}}$ , and the physics at such scale may be calculated with a perturbative expansion in  $\alpha_s$ . The relative momentum or “soft” scale, proportional to the inverse size of the system, may be a perturbative ( $\gg \Lambda_{\text{QCD}}$ ) or a non-perturbative scale ( $\sim \Lambda_{\text{QCD}}$ ) depending on the physical systems. Finally, only for  $t\bar{t}$  threshold states the binding energy, i.e. the “ultrasoft” scale, may still be perturbative. Heavy quark-antiquark states are thus an ideal

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and to some extent unique laboratory where our understanding of nonperturbative QCD, its interplay with perturbative QCD and the behaviour of the perturbative series in the bound state may be tested and understood in a controlled framework. This has been so historically, when more than 30 years ago the discovery of the  $J/\psi$  with its small width (controlled by  $\alpha_s$  at the mass scale) acted as an additional confirmation of the QCD asymptotic freedom idea. It is even more the case today for two reasons. First, remarkable theoretical progress has been achieved both in the formulation of nonrelativistic effective field theories (NR EFTs) for bound states of two heavy quarks [2] and in the lattice calculation of nonperturbative matrix elements. Second, the last few years have witnessed a kind of New Quarkonium Revolution in experiments with the discovery of more new states, decays and production mechanisms in the last three years [3, 4, 5] than in the entire previous thirty years.

The progress in our understanding of NR EFTs makes it possible to move beyond phenomenological models and to provide a systematic description inside QCD of all aspects of heavy-quarkonium physics. On the other hand, the recent progress in the measurement of several heavy-quarkonium observables makes it meaningful to address the problem of their precise theoretical determination. As we will discuss in the following sections, in this situation heavy quarkonium becomes a very special and relevant system to advance our theoretical understanding of the strong interactions, also in special environments (e.g. quarkonium in media) and in several production mechanisms, as well as our control of some parameters of the Standard Model [3, 4].

## 2. Theory Developments: Effective Field Theories

The modern approach to heavy quarkonium is provided by NR EFTs [2]. The idea is to take advantage of the existence of a hierarchy of scales to substitute QCD with simpler but equivalent NR EFTs. A hierarchy of EFTs may be constructed by systematically integrating out modes associated to high energy scales not relevant for the quarkonium system. Such integration is made in a matching procedure that enforces the complete equivalence between QCD and the EFT at a given order of the expansion in  $v$  ( $v^2 \sim 0.1$  for  $b\bar{b}$ ,  $v^2 \sim 0.3$  for  $c\bar{c}$ ,  $v \sim 0.1$  for  $t\bar{t}$ ). The EFT realizes a factorization at the Lagrangian level between the high energy contributions carried by matching coefficients and the low energy contributions carried by the dynamical degrees of freedom. The Poincaré symmetry remains intact at the level of the NR EFT in a nonlinear realization that imposes exact relations among the EFT matching coefficients [6].

### 2.1. Non Relativistic QCD (NRQCD)

NRQCD is the EFT for two heavy quarks that follows from QCD by integrating out the hard scale  $m$  [7, 8]. Only the upper (lower) components of the Dirac fields matter for quarks (antiquarks) at energies lower than  $m$ .

The Lagrangian is organized as an expansion in  $v$  and  $\alpha_s(m)$  of the type:

$$\mathcal{L}_{\text{NRQCD}} = \sum_n c_n(m, \mu) \times O_n/m^n, \quad (1)$$

$\mu$  being the EFT factorization scale. The NRQCD matching coefficients  $c_n$  are series in  $\alpha_s$  and encode the high energy contributions. The low energy operators  $O_n$  are constructed out of two or four heavy quark/antiquark fields plus gluons. The operators bilinear in the fermion (or in the antifermion) fields are the same that can be obtained from a Foldy–Wouthuysen transformation of the QCD Lagrangian. Four fermion operators have to be added. Matrix elements of  $O_n$  depend on the scales  $\mu$ ,  $mv$ ,  $mv^2$  and  $\Lambda_{\text{QCD}}$ . Hence, operators are counted in powers of  $v$ . The imaginary part of the coefficients of the 4-fermion operators contains the information on heavy quarkonium annihilation. The NRQCD heavy quarkonium Fock state is given by a series of terms, increasingly subleading, where the leading term is a  $Q\bar{Q}$  in a color singlet state and the first correction, suppressed in  $v$ , comes from a  $Q\bar{Q}$  in an octet state plus glue. NRQCD is suitable for studies of spectroscopy (on the lattice), inclusive decays and production.

### 2.2. potential Non Relativistic QCD (pNRQCD)

In NRQCD, the soft and ultrasoft scales are dynamical. This results in an ambiguous power counting and in calculations still complicated by the presence of two scales. In the last decade, the problem of systematically treating the remaining dynamical scales in an EFT framework has been addressed by several groups [9] and has now reached a good level of understanding. So one can go down one step further and integrate out also the soft scale, matching to the lowest energy EFT that can be introduced for quarkonia, where only the ultrasoft degrees of freedom remain dynamical. Potential Non Relativistic QCD (pNRQCD) [10, 11, 2] is the EFT for two heavy quark systems that follows from NRQCD by integrating out the soft scale  $mv$ . The leading order equation of motion is the Schrödinger equation whose potential is a matching coefficient of pNRQCD.

Depending on the size of the quarkonium we may distinguish two situations. When  $mv^2 \gtrsim \Lambda_{\text{QCD}}$  we speak about weakly coupled pNRQCD because the soft scale is perturbative and the matching from NRQCD to pNRQCD may be performed in perturbation theory. The degrees of freedom are  $Q\bar{Q}$  states, singlet and octet in color, and (ultrasoft) gluons, which

are multipole expanded. The Lagrangian is given by an expansion of the type

$$\sum_{k,n} \frac{c_k(m, \mu)}{m^k} \times V_n(r, \mu', \mu) \times O_{n,k} r^n, \quad (2)$$

$V_n$  being the pNRQCD matching coefficients. The bulk of the interaction is carried by potential-like terms, but non-potential interactions, associated with the propagation of low energy degrees of freedom are present as well and start to contribute at NLO in the multipole expansion. They are typically related to nonperturbative effects [11]. Matrix elements of  $O_{n,k}$  depend on the scales  $\mu'$ ,  $mv^2$  and  $\Lambda_{\text{QCD}}$ .

When  $mv \sim \Lambda_{\text{QCD}}$  we speak about strongly coupled pNRQCD because the soft scale is nonperturbative and the matching from NRQCD to pNRQCD may not be performed in perturbation theory. The matching coefficients may be obtained in the form of expectation values of gauge invariant Wilson loop operators. In this case, away from threshold (when heavy-light meson pair and heavy hybrids develop a mass gap of order  $\Lambda_{\text{QCD}}$  with respect to the energy of the  $Q\bar{Q}$  pair), the quarkonium singlet field  $S$  is the only low energy dynamical degree of freedom in the pNRQCD Lagrangian (neglecting pions and other Goldstone bosons), which reads [12, 13, 2]:

$$\mathcal{L}_{\text{pNRQCD}} = S^\dagger \left( i\partial_0 - \frac{\mathbf{p}^2}{2m} - V_S(r) \right) S. \quad (3)$$

The potential  $V_S(r)$  is a series in the expansion in the inverse of the quark masses; static,  $1/m$  and  $1/m^2$  terms have been calculated, see [12, 13]. They involve NRQCD matching coefficients and low energy nonperturbative parts given in terms of Wilson loops and field strengths insertions in the Wilson loop. In this regime, from pNRQCD we recover the quark potential singlet model. However, here the potentials are calculated from QCD by nonperturbative matching. Their evaluation requires calculations on the lattice [14] or in QCD vacuum models [1, 15].

Along the same lines also pNRQCD for  $QQ$  states (relevant for doubly charmed baryons) [16, 17] and for  $QQQ$  states [16] has been constructed in the two regimes. Recently the first lattice calculation of the  $QQq$  potential has appeared [18].

### 2.3. Present Reach of Theory

The physical reach of NRQCD and pNRQCD (combined for some processes with soft collinear effective theory, SCET) for heavy quarkonium includes spectra, inclusive and semi-inclusive decays, transitions and production.

For what concerns spectra and decays, the recent understanding of the renormalization group logarithm resummation for correlated scales [19] and of renormalon subtraction (for a review see [2]) has impressively extended the reach of QCD higher order perturbative calculations. Moreover, the reduction in the number of nonperturbative matrix elements obtained at the level of pNRQCD has greatly enhanced the predictive power of the theory [2].

Among recent applications, we would like to recall: the precise determination of the masses of the  $b$  and  $c$  quark from quarkonium with an error better than 50 MeV (see e.g. the average masses and the errors given in [3], see also [2, 20]); the recent extraction of  $\alpha_s$  from  $\Upsilon(1S)$  decay resulting in  $\alpha_s(M_Z) = 0.119^{+0.006}_{-0.005}$  in agreement with the central value of the PDG and with competitive errors [21]; studies of  $t\bar{t}$  production near threshold presently accurate at NNLO in perturbation theory with the complete logarithm resummation at NLL [22, 23]; a full understanding of the photon spectrum of radiative  $\Upsilon(1S)$  decays measured by CLEO [24].

For the implications of quarkonium on the search for new physics see [25].

In the following, we will briefly summarize the present theoretical status for few selected examples.

### 3. Potentials and Static Energy

The  $Q\bar{Q}$  potential is a Wilson matching coefficient of pNRQCD obtained by integrating out all degrees of freedom but the ultrasoft ones. If the quarkonium system is small, the soft scale is perturbative and the potential can be entirely calculated in perturbation theory [2]. It undergoes renormalization, develops a scale dependence and satisfies renormalization group equations, which eventually allow to resum potentially large logarithms. The static singlet potential is known at three loops apart from the constant term. The first logarithm related to ultrasoft effects arises at three loops. Such logarithm at N<sup>3</sup>LO and the single logarithm at N<sup>4</sup>LO may be extracted respectively from a one-loop and two-loop calculation in the EFT and have been calculated in [26, 27]. The static energy, given by the sum of a constant, the static potential and the ultrasoft corrections, is free from renormalon ambiguities. By comparing it (at NNLL order) with lattice calculations one sees that the QCD perturbative series converges very nicely to and agrees with the lattice data in the short range and that therefore no large linear (“stringy”) contribution to the static potential exists at short distances [28, 2].

#### 4. Perturbative calculations of Spectra

In the weak coupling, the soft scale is perturbative and the potentials are purely perturbative objects. Nonperturbative effects enter energy levels and decay widths calculations in the form of local or nonlocal condensates [29]. We still lack a precise and systematic knowledge of such nonperturbative purely glue dependent objects. It would be important to have for them lattice determinations or data extraction (see e.g. [30]). The leading electric and magnetic nonlocal correlators may be related to the gluelump masses [11] and to some existing lattice (quenched) determinations [2].

However, since the nonperturbative contributions are suppressed in the power counting it is possible to obtain good determinations of the masses of the lowest quarkonium resonances with purely perturbative calculations in the cases in which the perturbative series converges well (i.e. after the appropriate subtractions of renormalons have been performed and large logarithms have been resummed). In this framework, power corrections are unambiguously defined. Renormalon subtraction has been exploited in [31] to get a prediction of the  $B_c$  mass. The NNLO calculation with finite charm mass effects [32] predicts a mass of 6307(17) MeV that matches well the CDF measurement [33] and the lattice determination [34]. The same procedure seems to work at NNLO even for higher states (inside larger theoretical errors) [32]. Including logarithm resummation at NLL, it is possible to obtain a prediction for the mass of the  $\eta_b$ , which is  $9421 \pm 11_{-8}^{+9}$  MeV and for the  $B_c$  hyperfine separation,  $\Delta = 65 \pm 24_{-16}^{+19}$  MeV [35]. A NLO calculation reproduces in part the  $1P$  fine splitting [36].

#### 5. Lattice calculations of Potentials and Spectra

Traditionally NRQCD lattice calculations have been used to obtain the spectrum of the low lying  $b\bar{b}$  and  $c\bar{c}$  states. However the difficulty of the calculation of the NRQCD matching coefficients in the lattice scheme combined with the problem of the nonperturbative renormalization of the zeroth order NRQCD Lagrangian have in part hampered this approach. Recent unquenched results exist for  $b\bar{b}$  (with tree level matching coefficients) [38] while for the  $c\bar{c}$  the current trend is to use unquenched relativistic actions and anisotropic lattices [39].

In strongly coupled pNRQCD, the energy spectrum is obtained by solving the Schrödinger equation (3) with the potentials given in terms of the NRQCD matching coefficients times expectation values of Wilson loops with field strength insertions to be calculated on the lattice. Recently the  $1/m$  potential and the spin dependent and “velocity” dependent potentials at order  $1/m^2$  have been calculated on the lattice with unprecedented precision [37]. In the long range, the spin-orbit potentials show, for the first

time, deviations from the flux-tube picture of chromoelectric confinement. Since a fully consistent renormalization of the EFT operators is still missing in the lattice analysis, it may be premature to draw any definitive conclusion. However, progress has been made recently in this direction. In [45], the nonperturbative renormalization of the chromomagnetic operator in the Heavy Quark Effective Theory, which crucially enters in all spin-dependent potentials, has been performed for the first time.

The relations among the potentials imposed in pNRQCD by Poincaré invariance [6], have been checked on the lattice at the few percent level.

The zeroth order pNRQCD Lagrangian is renormalizable. Hence, pNRQCD may be well suited for direct lattice evaluation of quarkonium correlation functions.

## 6. Quarkonium Decays and Transitions

Expressions for inclusive electromagnetic and hadronic quarkonium decays are now known at order  $v^7$  in NRQCD [43, 44]. The matching coefficients are known at different accuracy in the  $\alpha_s$  expansion, for a review see [42]. At the moment, specific problems for phenomenological applications arise from the proliferation in the number of unknown nonperturbative matrix elements in NRQCD and the bad convergence of the perturbative series of some NRQCD matching coefficients. Only few NRQCD matrix elements have been calculated on the lattice up to now (see e.g. [46]). A significant reduction in the number of nonperturbative operators for inclusive decays is achieved in strongly coupled pNRQCD, where the NRQCD decay matrix elements factorize in a part, which is the wave function in the origin squared (or its derivatives), and in a part which contains gluon tensor-field correlators [30, 40, 41].

For the lowest resonances, inclusive decay widths are given in weakly coupled pNRQCD by a convolution of perturbative corrections and non-local nonperturbative correlators. The perturbative calculation embodies large contributions and requires the resummation of large logarithms (see e.g. A. Pineda in [4]). Recently, higher order contributions to quarkonium production and annihilation have been obtained [47].

Allowed magnetic dipole transitions between  $c\bar{c}$  and  $b\bar{b}$  ground states have been considered in pNRQCD at NNLO in [48]. The results are:  $\Gamma(J/\psi \rightarrow \gamma \eta_c) = (1.5 \pm 1.0) \text{ keV}$  and  $\Gamma(\Upsilon(1S) \rightarrow \gamma \eta_b) = (k_\gamma/39 \text{ MeV})^3 (2.50 \pm 0.25) \text{ eV}$ , where the errors account for uncertainties coming from higher-order corrections. The width  $\Gamma(J/\psi \rightarrow \gamma \eta_c)$  is consistent with the PDG value. Concerning  $\Gamma(\Upsilon(1S) \rightarrow \gamma \eta_b)$ , a photon energy  $k_\gamma = 39 \text{ MeV}$  corresponds to a  $\eta_b$  mass of 9421 MeV. The pNRQCD calculation features a small quarkonium magnetic moment (in agreement with a recent lattice cal-

ulation [49]) and the interesting fact, related to the Poincaré invariance of the NR EFT, that M1 transition of the lowest quarkonium states at relative order  $v^2$  are completely accessible in perturbation theory [48].

## 7. Quarkonium Production

Although a formal proof of the NRQCD factorization formula for heavy quarkonium production has not yet been obtained, NRQCD factorization has proved to be successful to explain a variety of quarkonium production processes (for a review see e. g. the production chapter in [3] and the more recent review [53]). In the last years, there has been progress toward an all order proof. In [54], it has been shown that a necessary condition for factorization to hold at NNLO is that the conventional octet NRQCD production matrix elements must be redefined by incorporating Wilson lines that make them manifestly gauge invariant. Differently from decay processes, a pNRQCD treatment does not exist so far for quarkonium production. In the last years, two main problems have plagued our understanding of heavy quarkonium production. The first BELLE and BABAR measurements of the cross section  $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$  were about one order of magnitude above theoretical expectations. Triggered by this, some errors have been corrected in some of the theoretical determinations, and, more relevant, NLO corrections in  $\alpha_s$  and in  $v^2$  have been calculated and some class of relativistic corrections has been resummed [55, 56, 58, 59, 60]. For calculations in the framework of light cone see [61].

One can say that now the discrepancy between the QCD theoretical prediction for  $\sigma(e^+e^- \rightarrow J/\psi + \eta_c)$  and the experimental measurements has been resolved. However the discrepancy seems to survive for the inclusive production  $\sigma(e^+e^- \rightarrow J/\psi + c\bar{c})$  where relativistic corrections are tiny [62]. In addition, the latest data on charmonium and bottomonium polarization at Tevatron (Run II) [4] contradict the prediction of NRQCD with traditional power counting. Recently, singlet contributions to quarkonium hadroproduction have been calculated at NLO [63] and hadroproduction of heavy quarkonium in association with an additional heavy quark pair has been calculated at LO [64]. Both contributions turn out to be sizeable and tend to unpolarize the produced quarkonium. A modification of the NRQCD factorization approach for processes involving production in association with another heavy quark pair may be necessary [65].



## 8. Theory Open Challenges

### 8.1. Threshold States

For states near or above threshold a general systematic theoretical treatment has still to be developed. At the moment, the first preliminary studies of excited resonances on the lattice are just appearing [50, 51, 52] some of them being still quenched. Most of the existing analyses have therefore to rely on phenomenological models.

However, in some cases, a theoretical treatment based on an EFT approach has been developed. This is notably the case of the  $X(3872)$  in the molecular picture [66]. Most of the newly discovered states in the charmonium region lie close to threshold or over threshold. The confirmation of some of these new states would require a trustable calculation of individual contributions and interference terms in the total cross section. *It is high priority for theory to develop a systematical effective field theory approach to quarkonium states close to threshold and coupled to heavy-light mesons.*

### 8.2. Quarkonium at Finite $T$

Quarkonium suppression is believed to be a clean signal for quark gluon plasma formation in heavy ion collisions, to be, however, considered together with possible quarkonium recombination effects in the medium. An extensive literature in the field deals both with lattice calculations of the free energy of a quark-antiquark pair as well as with model calculations of the quark-antiquark correlators and spectral functions at finite  $T$  [67]. As a matter of fact, it has not yet been understood how to define the quark-antiquark static potential at finite  $T$ , even if recently some steps forward have been accomplished, obtaining a (complex-valued) potential from a perturbative calculation of the Wilson loop [68]. *It is a high priority for theory to develop an EFT systematical approach to quarkonium physics at finite  $T$ ,* where a potential may be clearly defined and calculations may be performed that include all the relevant low energy dynamical degrees of freedom.

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